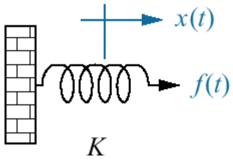
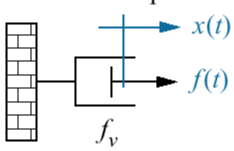
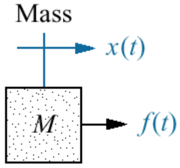
B) <u>Modelling of translational mechanical systems</u>

- **Like the electrical network, the mechanical systems have three passive, linear components: spring and mass (both are energy-storage elements) and viscous damper (energy-dissipative element)**

- **Both the energy-storage elements are analogous to the inductor and capacitor, while the energy-dissipative element is analogous to the electrical resistance in the electrical network.**

- **The translational mechanical relationships is shown in the following table:**

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)\,d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

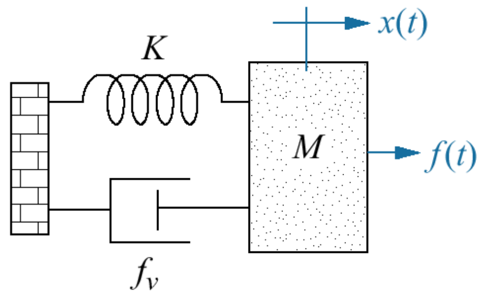
Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds^2/meter).

- **In the table, *K*, f_v and *M* are spring constant, coefficient of viscous friction, and mass, respectively.**

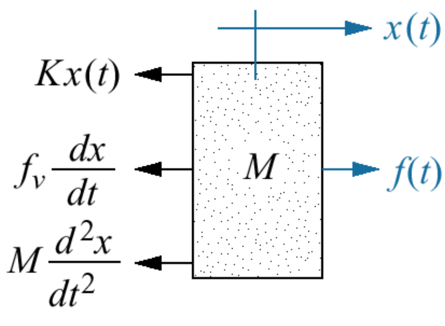
- The mechanical system just requires one differential equation, called the equation of motion to describe it.

- We use the free-body diagram and Newton's law to form a differential equation of motion by summing the forces and setting the sum equal to zero. Then, taking the Laplace Transform of the differential equation to get the transfer function.

- Example: Find the transfer function, *X(s)/F(s)*.



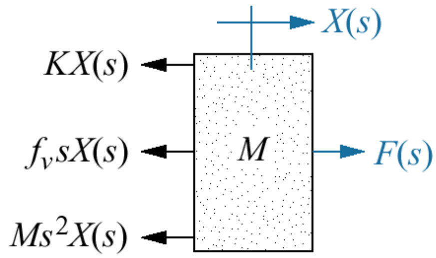
Draw the free-body diagram



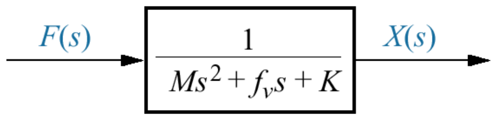
We can write the differential equation of motion

And taking the Laplace transform gives

Or, we can transform the free-body diagram into,



To yield,



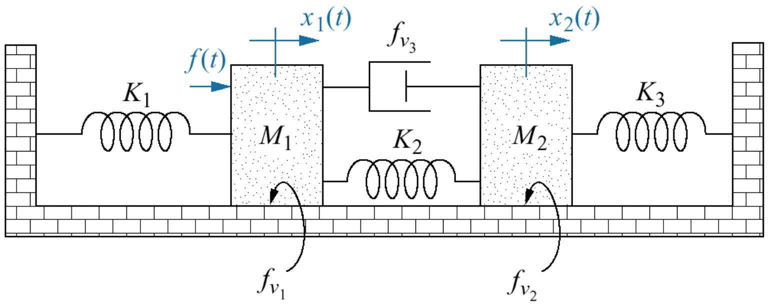
- **In mechanical systems, the number of equations of motion required is equal to the number of linearly independent motions.**

- **Hence, we can draw the free-body diagram for each point of motion and then use superposition.**

- **For each free-body diagram, hold all other points of motion still and find the forces acting on the body due only its own motion. Then, hold the body still and activate the other points of motion one at a time.**

- **Next, sum the forces on each body and set the sum to zero.**

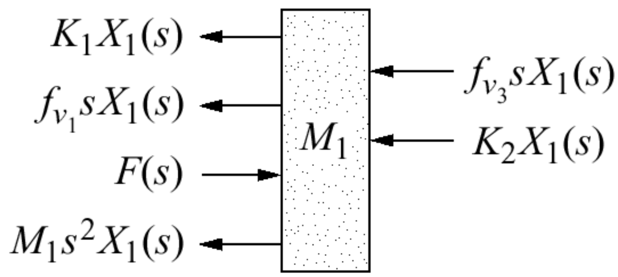
- **Example: Find the transfer function, $X_2(s)/F(s)$**



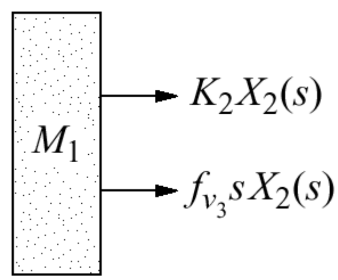
Two equations come from free-body diagrams of each mass

1. **The forces on M1 due to its own motion and the motion of M2 transmitted to M1 through the system.**

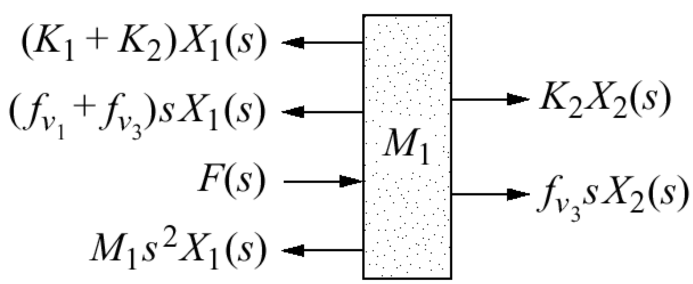
   If we hold M2 still and move M1 to the right,



   If we hold M1 still and move M2 to the right,



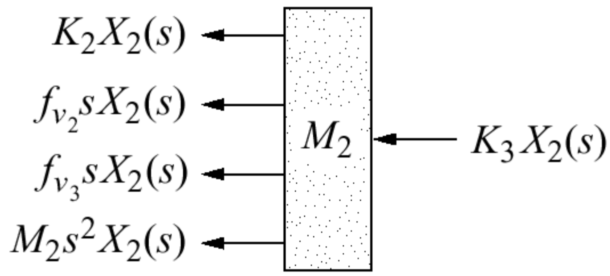
   The total force on M1 is the superposition, or sum, of forces above,



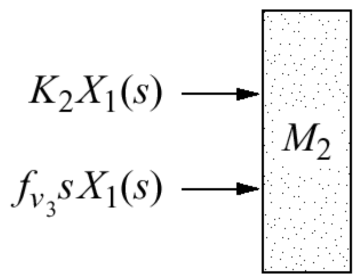
   Giving the equation of motion of

2. **The forces on M2 due to its own motion and the motion of M1 transmitted to M2 through the system.**

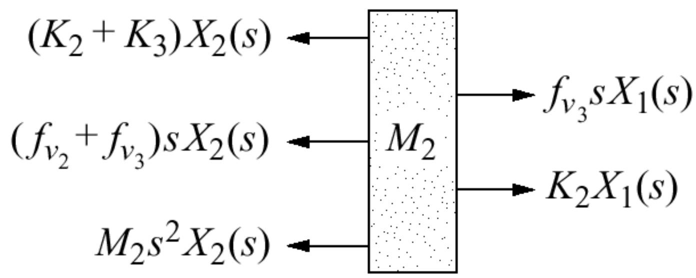
   If we hold M1 still and move M2 to the right,



   If we hold M2 still and move M1 to the right,



   The total force on M2 is the superposition, or sum, of forces above,



   Giving the equation of motion of

The Laplace transform of equations of motion can now be written as

And the transfer function is

$$\frac{X_2(s)}{F(s)} = G(s) = \frac{(f_{v3}s + K_2)}{\Delta}$$

where,

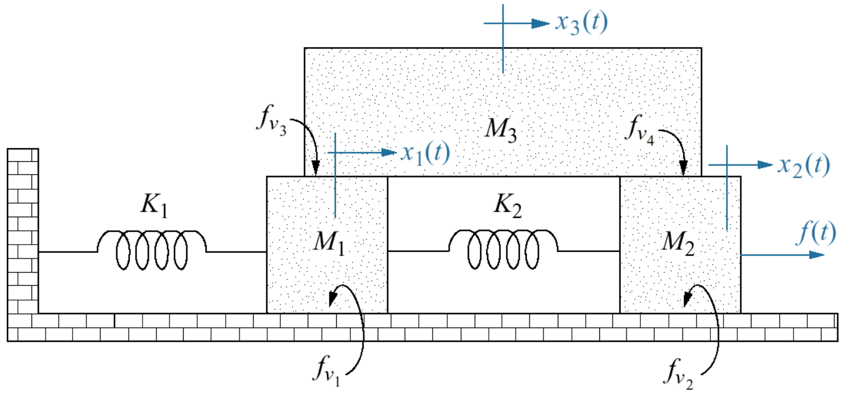
$$\Delta = \begin{vmatrix} \left[M_1 s^2 + (f_{v1} + f_{v3})s + (K_1 + K_2)\right] & -(f_{v3}s + K_2) \\ -(f_{v3}s + K_2) & \left[M_2 s^2 + (f_{v2} + f_{v3})s + (K_2 + K_2)\right] \end{vmatrix}$$

Notice that the form of equations is similar to the electrical mesh equations:

$$\begin{bmatrix} sum\, of\ impedences \\ connected\, to\, the \\ motion\, at\, x_1 \end{bmatrix} X_1(s) - \begin{bmatrix} sum\, of\ impedences \\ between\, x_1\ and\ x_2 \end{bmatrix} X_2(s) = \begin{bmatrix} sum\, of \\ applied \\ forces\, at\, x_1 \end{bmatrix}$$

$$-\begin{bmatrix} sum\, of\ impedences \\ between\, x_1\ and\ x_2 \end{bmatrix} X_1(s) + \begin{bmatrix} sum\, of\ impedences \\ connected\, to\, the \\ motion\, at\, x_2 \end{bmatrix} X_2(s) = \begin{bmatrix} sum\, of \\ applied \\ forces\, at\, x_2 \end{bmatrix}$$

- **Example: Write down the equations of motion**



- **Exercise: Find the transfer function, *$G(s)=X_2(s)/F(s)$* for the following translational system**