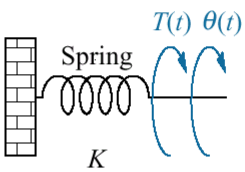
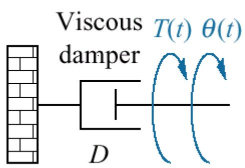
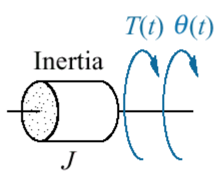
C) <u>Modelling of rotational mechanical systems</u>

- **Rotational mechanical systems are handled the same way as the translational mechanical systems, except that torque replaces force and angular displacement replaces translational displacement, while the mass is replaced by inertia.**

- **The rotational mechanical relationships is shown in the following table:**

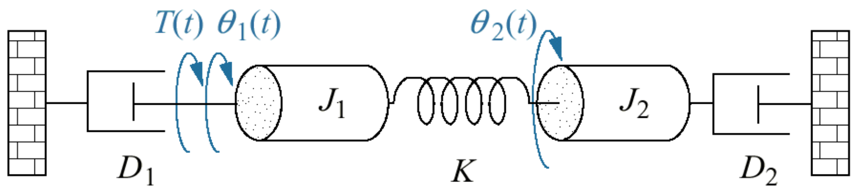
Component	Torque-angular velocity	Torque-angular displacement	Impedance $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)\,d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\dfrac{d\theta(t)}{dt}$	Ds
	$T(t) = J\dfrac{d\omega(t)}{dt}$	$T(t) = J\dfrac{d^2\theta(t)}{dt^2}$	Js^2

Note: The following set of symbols and units is used throughout this book: $T(t)$ = N-m (newton-meters), $\theta(t)$ = rad (radians), $\omega(t)$ = rad/s (radians/second), K = N-m/rad (newton-meters/radian), D = N-m-s/rad (newton-meters-seconds/radian), J = kg-m^2 (kilogram-meters^2 = newton-meters-seconds^2/radian).

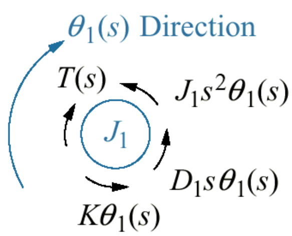
- **The values of *K, D* and *J* are called spring constant, coefficient of viscous friction and moment of inertia, respectively**

- **Writing the equation of motion for rotational systems is similar to writing them for the translational systems; the only difference is that the free-body diagram consists of torques rather than forces.**

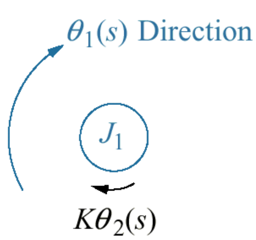
- **Example: Find the transfer function, $\theta_2(s)/T(s)$ for the following rotational system**



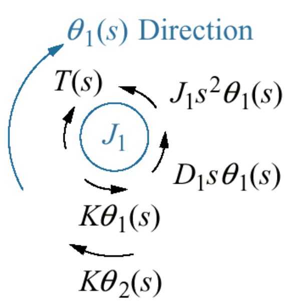
Torques on J1 due only to the motion of J1:



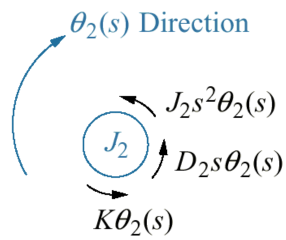
Torques on J1 due only to the motion of J2:



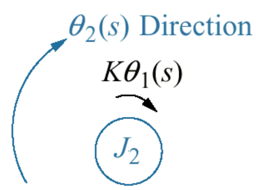
Final free-body diagram for J1:



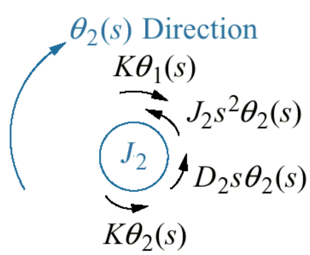
Torques on J2 due only to the motion of J2:



Torques on J2 due only to the motion of J1:



Final free-body diagram for J2:



The Laplace transform of equations of motion can now be written as

And the transfer function is

$$\frac{\theta_2(s)}{T(s)} = G(s) = \frac{\quad}{\Delta}$$

where,

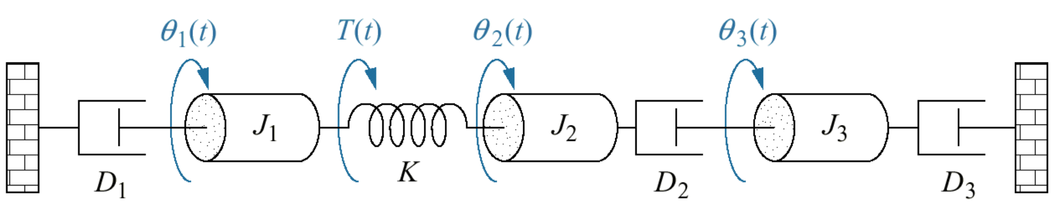
$$\Delta =$$

Notice that the well-known form of equations now is

$$\begin{bmatrix} sum\ of\ impedences \\ connected\ to\ the \\ motion\ at\ \theta_1 \end{bmatrix}\theta_1(s) - \begin{bmatrix} sum\ of\ impedences \\ between\ \theta_1\ and\ \theta_2 \end{bmatrix}\theta_2(s) = \begin{bmatrix} sum\ of \\ applied \\ torques\ at\ \theta_1 \end{bmatrix}$$

$$-\begin{bmatrix} sum\ of\ impedences \\ between\ \theta_1\ and\ \theta_2 \end{bmatrix}\theta_1(s) + \begin{bmatrix} sum\ of\ impedences \\ connected\ to\ the \\ motion\ at\ \theta_2 \end{bmatrix}\theta_2(s) = \begin{bmatrix} sum\ of \\ applied \\ torques\ at\ \theta_2 \end{bmatrix}$$

- **Example: Write the Laplace transform of the equations of motion for the following system**



- **Example: Find the transfer function, $G(s) = \theta_2(s)/T(s)$**