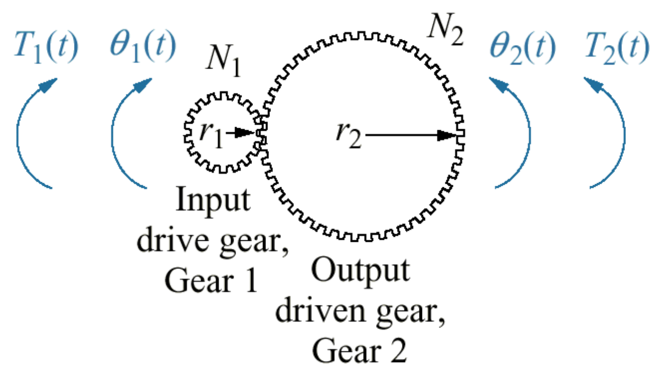
D) **Modelling of rotational mechanical systems with gears**

- **Gears provide mechanical advantage to rotational systems**

- **A simple example is when one riding a bicycle with a 5- or 10-speed gears, esp. when going uphill, downhill and straightaway (shifting gears for more torque and less speed, and for less torque and more speed)**

- **Assuming that there is no backlash on the gears, a interaction between two gears can be given as follows:**



- **Input drive gear giving a torque, *$T_1(t)$*, and the output drive gear producing a torque, *$T_2(t)$*. Hence the relationship between the the rotation of gear 1 and gear 2 can be established.**

- **The distance traveled along each gear's circumference is the same:**

$$r_1\theta_1 = r_2\theta_2$$

or

$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}$$

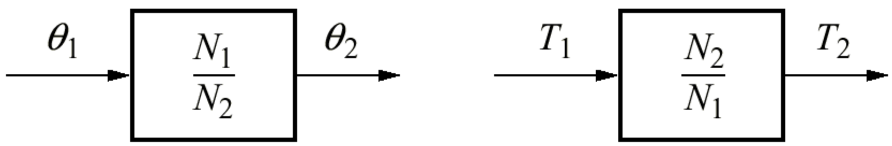
- **Assuming that there is no energy lost or absorbed, the energy into gear 1 is equal to the energy out of gear 2,**

$$T_1\theta_1 = T_2\theta_2$$

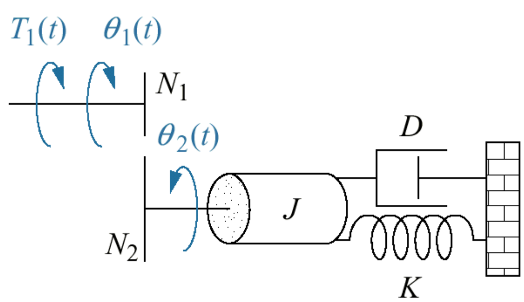
- **Hence, we have**

$$\frac{T_2}{T_1} = \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1} = \frac{r_2}{r_1}$$

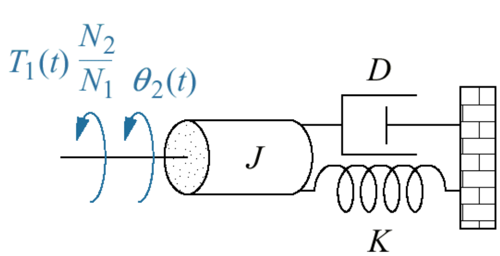
where, the torques are directly proportional to the ratio of the number of teeth.



- **Look at an example of a mechanical system which shows the gears driving a rotational inertia, spring and viscous damper:**



We can represent the system with an equivalent system at θ_1 without gears: T_1 can be reflected to the output by multiplying the gear ratio,



$$\left(Js^2 + Ds + K\right)\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

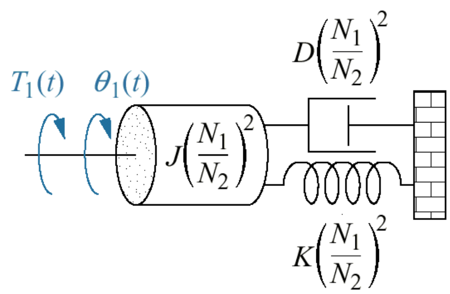
Converting θ_2 into an equivalent θ_1 gives

$$\left(Js^2 + Ds + K\right)\frac{N_1}{N_2}\theta_1(s) = T_1(s)\frac{N_2}{N_1}$$

After simplification gives

$$\left[J\left(\frac{N_1}{N_2}\right)^2 s^2 + D\left(\frac{N_1}{N_2}\right)^2 s + K\left(\frac{N_1}{N_2}\right)^2\right]\theta_1(s) = T_1(s)$$

which suggests the equivalent system at the input and without gears



- **Generalized result: we can state that *the rotational mechanical impedances can be reflected through gear trains by multiplying the mechanical impedance by the ratio***

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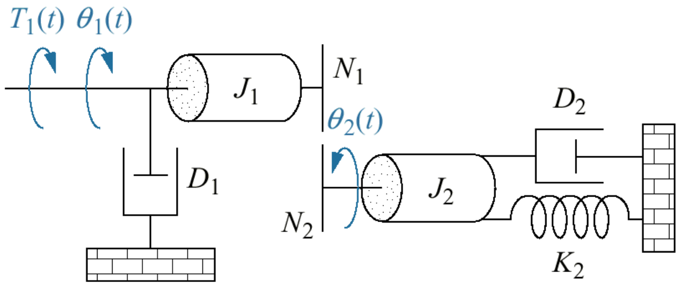
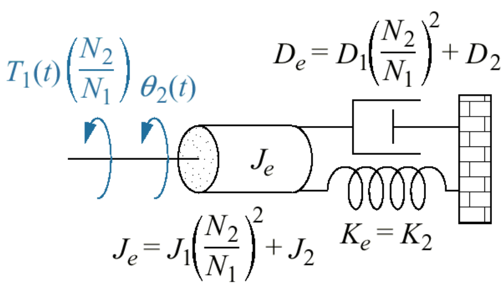
$$\left(\frac{\text{Number of teeth of gear on destination shaft}}{\text{Number of teeth of gear on source shaft}}\right)^2$$

- **Example: Find the transfer function, $\theta_2(s)/T_1(s)$, for the following system**

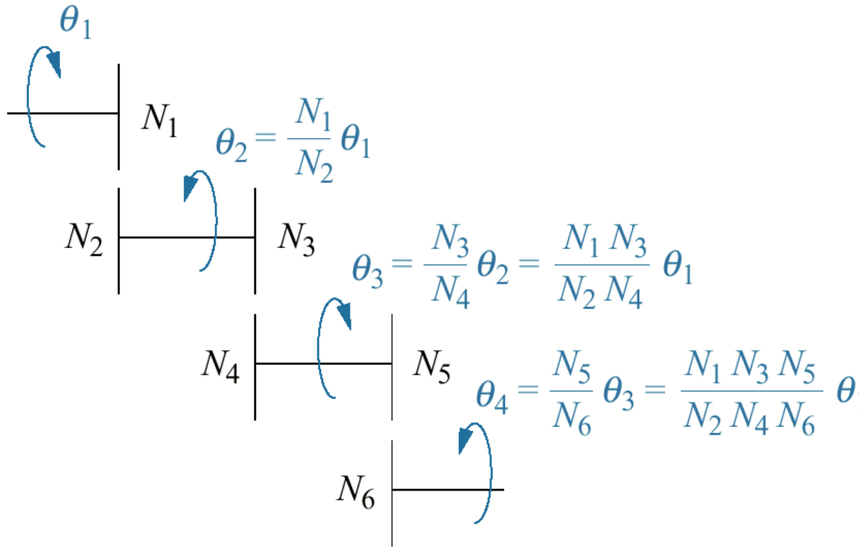


Reflect impedances to $\theta_2(s)$ gives

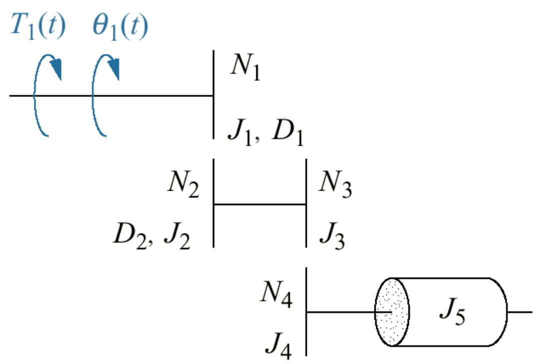


Hence, the transfer function is

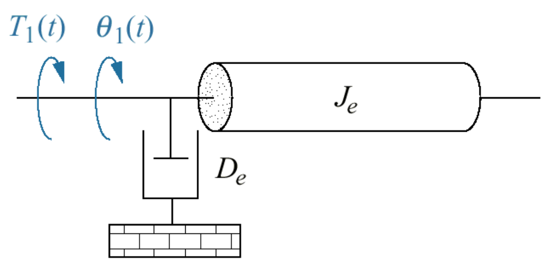
- **For gear trains, we can conclude that the equivalent gear ratio is the product of the individual gear ratio (with no losses)**



- **Find the transfer function, $\theta_1(s)/T_1(s)$, for the following system**



Reflecting all impedances to the input shaft gives



$$J_e = J_1 + (J_2 + J_3)\left(\frac{N_1}{N_2}\right)^2 + (J_4 + J_5)\left(\frac{N_1 N_3}{N_2 N_4}\right)^2$$

$$D_e = D_1 + D_2\left(\frac{N_1}{N_2}\right)^2$$

- **Find the transfer function, $\theta_2(s)/T(s)$, for the following rotational mechanical system with gears**