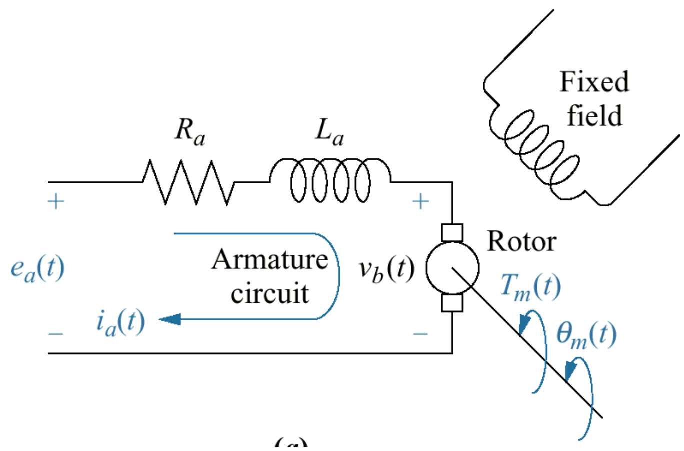
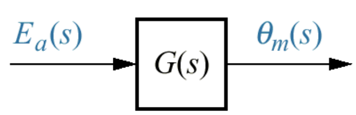
E) **Modelling of electromechanical system**

- **An electromechanical system is a system that gives a displacement output from a voltage input, eg, motor. (mechanical output generated by electrical input.**

- **Derivation of an armature-controlled dc servomotor transfer function as follows:**



We would like to represent the above system with the following block diagram:



A magnetic field is developed by a fixed field electromagnet.

The armature circuit passes current, $i_a(t)$ through the magnetic field in the motor, producing a force

$$F = Bli_a(t)$$

where, **B = magnetic field strength**

l = length of the conductor

The resultant torque turns the rotor.

Voltage across the rotor terminal is proportional to the rotor speed, and is given by

$$v_b = K_b \frac{d\theta_m(t)}{dt} \qquad \textbf{(1)}$$

known as the electromotive force (back emf), where

K_b **is the back emf constant**

$\frac{d\theta_m(t)}{dt}$ **is the angular velocity of the motor**

Taking LT of (1) gives,

$$V_b(s) = K_b s\theta_m(s) \qquad \textbf{(2)}$$

Looking at the armature circuit,

$$R_a I_a(s) + L_a s I_a(s) + V_b(s) = E_a(s) \qquad \textbf{(3)}$$

The torque developed by the motor is proportional to the armature current,

$$T_m(s) \propto I_a(s)$$

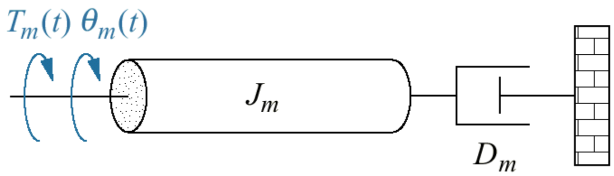
$$\Rightarrow \quad T_m(s) = K_t I_a(s) \qquad \textbf{(4)}$$

where K_t **is the motor torque constant (depends on the motor and magnetic field circuit)**

Substituting (2) and (4) into (3) gives

$$\frac{(R_a + L_a s)T_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s) \qquad \textbf{(5)}$$

Now, we have to find $T_m(s)$ in terms of $\theta_m(s)$ so that we can separate the input and the output variables and obtain the transfer function.



The above figure shows the equivalent mechanical load on the motor:

J_m is the equivalent at the armature and the load

D_m is the equivalent viscous damping at the armature and the load

and the relationship is given by

$$T_m(s) = \left(J_m s^2 + D_m s\right)\theta_m(s) \qquad \textbf{(6)}$$

Substituting (6) into (5) gives

$$\frac{(R_a + L_a s)\left(J_m s^2 + D_m s\right)\theta_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s) \qquad \textbf{(7)}$$

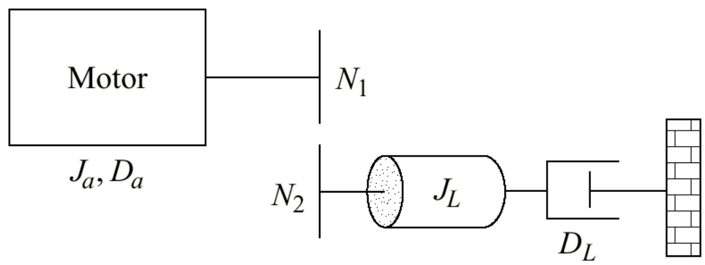
Assuming that $L_a << R_a$, which is usual for a dc motor, gives

$$\left[\frac{R_a}{K_t}\left(J_m s + D_m\right) + K_b\right] s\theta_m(s) = E_a(s) \qquad \textbf{(8)}$$

After simplification, the desired transfer function is

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/\left(R_a J_m\right)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]} \qquad \textbf{(9)}$$

- **We can evaluate the mechanical and electrical constants:**
  - **For the mechanical constants,**



Reflecting the load to the armature gives

$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2 \quad \textbf{(10)}$$

$$D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2 \quad \textbf{(11)}$$

  - **For the electrical constants,**

From (5), when $L_a \approx 0$**,**

$$\frac{R_a}{K_t}T_m(s) + K_b s\theta_m(s) = E_a(s) \quad \textbf{(11)}$$

Taking inverse Laplace Transform on (11) gives

$$\frac{R_a}{K_t}T_m(t) + K_b\omega_m(t) = e_a(t) \quad \textbf{(12)}$$

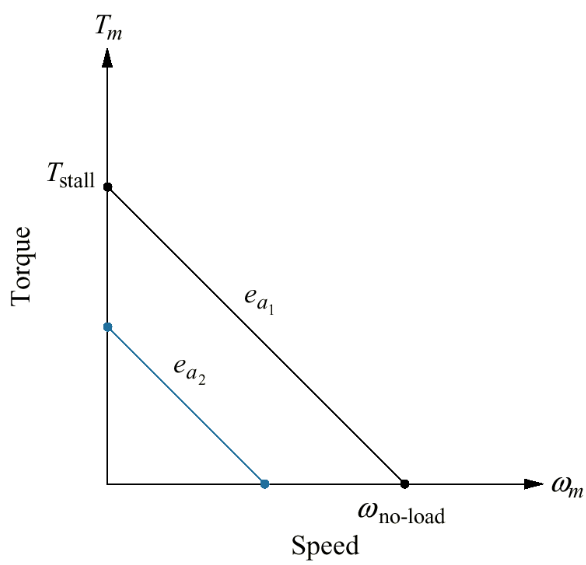
where $\omega_m(t) = \frac{d\theta(t)}{dt}$

Applying a dc voltage on (12) will turn the motor at a constant velocity and a constant torque,

$$\frac{R_a}{K_t} T_m + K_b \omega_m = e_a \qquad \textbf{(13)}$$

which gives a straight line equation,

$$T_m = -\frac{K_b K_t}{R_a} \omega_m + \frac{K_t}{R_a} e_a \qquad \textbf{(14)}$$



The torque-speed curve

When the angular velocity is zero, the value of the torque, called the stall torque, is

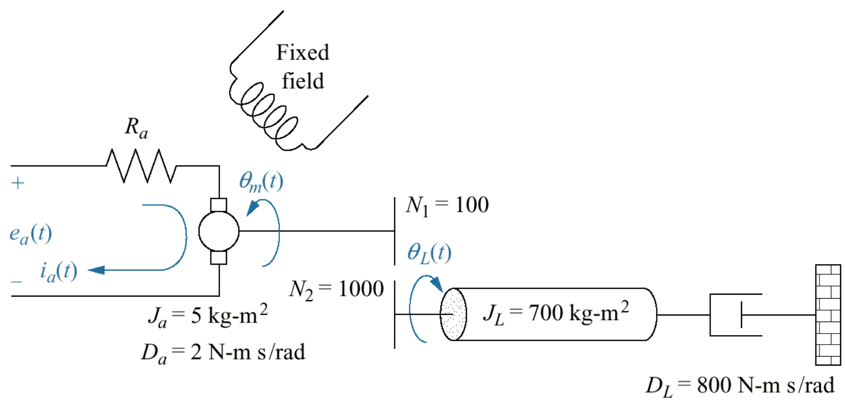
$$T_{stall} = \frac{K_t}{R_a} e_a \qquad \textbf{(15)}$$

When the torque is zero, the angular velocity, called the no-load speed, is

$$\omega_{no-load} = \frac{e_a}{K_b} \qquad \textbf{(16)}$$

The electrical constants, can be found using a dynamometer test of the motor, which would yield the T_{stall} and $\omega_{no\text{-}load}$ for a given e_a

- **Example: Given a system with a torque-speed curve below, find the transfer function,** $\theta_L(s)/E_a(s)$



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