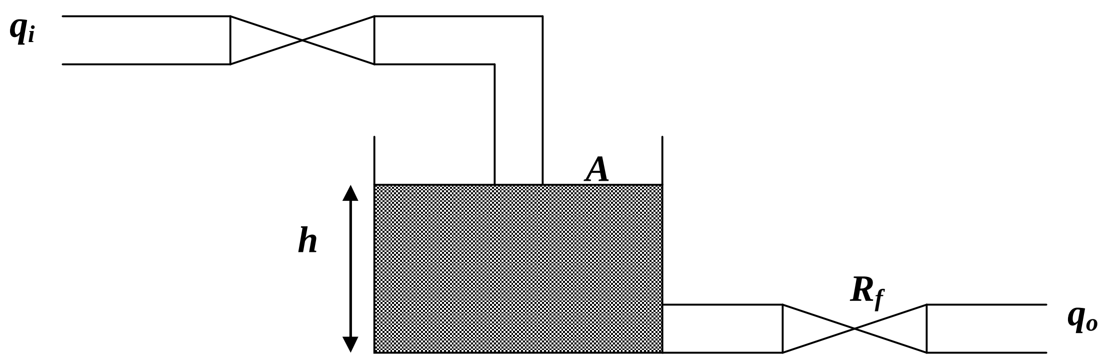
F) Modelling of liquid level systems

- **Liquid level systems are physical systems that involve liquids (hydraulics) or gas (pneumatics) as the medium of transmission.**

- **In this course, we will only look at hydraulic systems where the liquid used is not compressible.**

- **We also consider only the laminar flow (not turbulent flow)**

- **Consider the liquid level system below:**



It is desired to control the level of water in the tank, *h*, by controlling the input flow rate, q_i.

The output valve has a resistance of R_f and the output flow rate is q_o.

We need to find the transfer function, H(s)/Q_i(s).

From the law of the conservation of mass,

- **inflow rate – outflow rate = increase in mass**

$$q_i - q_o = A\frac{dh}{dt}$$

where, *A* is the surface area of the tank

- **The liquid level in the tank, *h*, is proportional to the output flow rate of the liquid, ie,**

$$h \propto q_o$$

$$h = R_f q_o$$

$$\Rightarrow \qquad q_o = \frac{h}{R_f}$$

- **Hence,**

$$q_i - \frac{h}{R_f} = A\frac{dh}{dt}$$

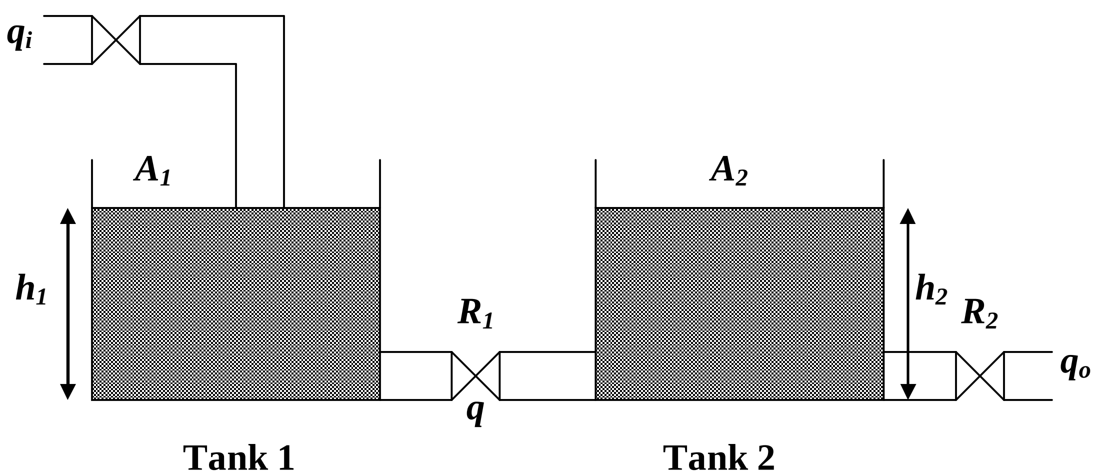
- **Taking the Laplace transform, gives**

$$Q_i(s) - \frac{H(s)}{R_f} = AsH(s)$$

$$\therefore \quad \frac{H(s)}{Q_i(s)} = \frac{R_f}{AR_f s + 1}$$

- **Example:**

Find the transfer function, $H_2(s)/Q_i(s)$ for the following system



- **The output flow rate for Tank 1 is given by:**

$$h_1 - h_2 = R_1 q \qquad \textbf{(1)}$$

$$q = \frac{h_1 - h_2}{R_1}$$

- **For tank 2,**

$$h_2 = R_2 q_o \qquad \textbf{(2)}$$

- **Using the law of conservation of mass,**

$$q_i - q = A_1 \frac{dh_1}{dt} \qquad \textbf{(3)}$$

$$q - q_o = A_2 \frac{dh_2}{dt} \qquad \textbf{(4)}$$

- **Taking Laplace Transform for (1), (2), (3) and (4),**

$$H_1(s) = R_1 Q(s) + H_2(s) \qquad \textbf{(5)}$$

$$Q_o(s) = \frac{H_2(s)}{R_2} \qquad \textbf{(6)}$$

$$Q_i(s) - Q(s) = A_1 s H_1(s) \qquad \textbf{(7)}$$

$$Q(s) - Q_o(s) = A_2 s H_2(s) \qquad \textbf{(8)}$$

- **Substitute (5) into (7) for *$H_1(s)$* and (6) into (8) for *$Q_o(s)$*, gives, respectively,**

$$Q_i(s) - Q(s) = A_1 s\left[R_1 Q(s) + H_2(s)\right] \qquad \textbf{(9)}$$

$$Q(s) - \frac{H_2(s)}{R_2} = A_2 s H_2(s) \qquad \textbf{(10)}$$

- **Eliminating *$Q(s)$* in (9) and (10) gives,**

$$\frac{H_2(s)}{Q_i(s)} = \frac{R_2}{A_1 R_1 A_2 R_2 s^2 + (A_1 R_1 + A_2 R_2 + A_1 R_2)s + 1}$$