F) Steady state errors

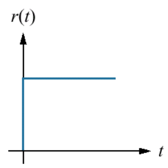
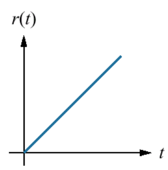
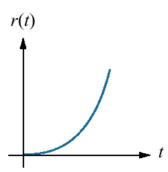
- In steady state response, we are trying to investigate a system's behaviour as t is approaching infinity.

- The quantity that we are investigating at steady state response is the *steady-state error*, e_{ss}, of a system.

- *The steady-state error* is defined as the difference between the input and the output for a prescribed test input as $t \rightarrow$ infinity.

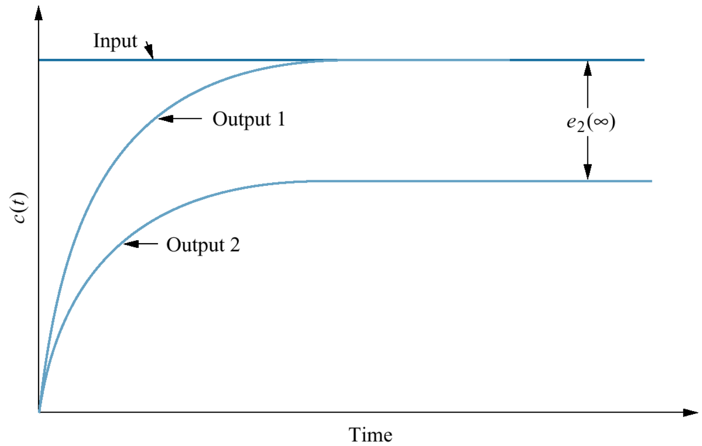
$$E(s) = \text{input} - \text{output}$$
$$= R(s) - C(s)$$

- Some common test inputs used for steady-state error analysis and design are:

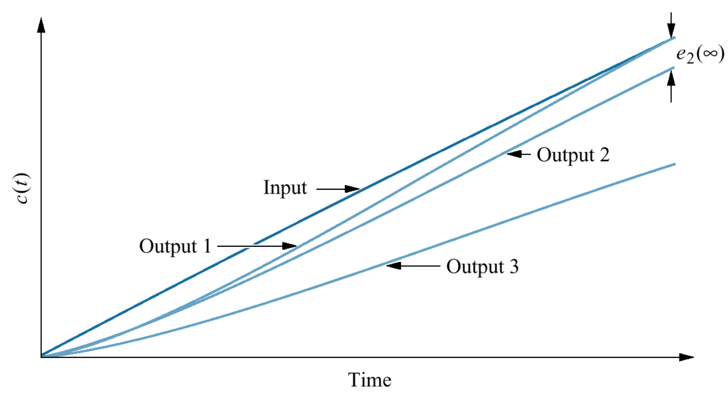
Waveform	Name	Physical interpretation	Time function	Laplace transform
	Step	Constant position	1	$\frac{1}{s}$
	Ramp	Constant velocity	t	$\frac{1}{s^2}$
	Parabola	Constant acceleration	$\frac{1}{2}t^2$	$\frac{1}{s^3}$

- Steady-state error analysis only applicable to stable systems, as the unstable systems represent the loss of control in steady state.

- **Example for evaluating steady-state error:**

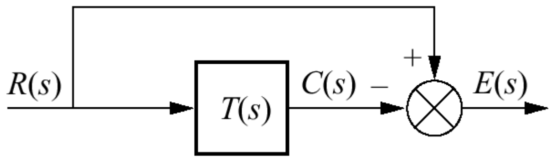


- **For a step input, output 1 has zero steady-state error, while output 2 has a finite steady-state error.**

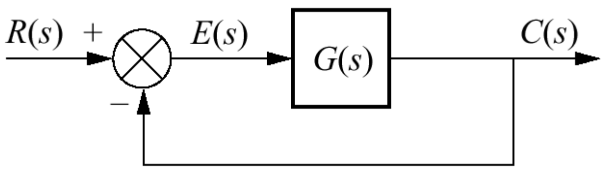


- **Similarly, for a ramp input, output 1 has zero steady-state error, output 2 has a finite steady-state error (the difference is measured vertically), while output 3 has an infinite value of steady-state error.**

- **The steady-state error is the difference between the input and output, hence we assume a closed loop transfer function, *T(s)*. The general representation of steady-state error is**

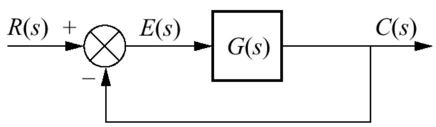


- **For a unity feedback system, the steady-state is represented by**



- **The steady-state error for <u>a unity feedback</u> system:**

  - **Consider the following block diagram,**



  - **The closed loop transfer function is**

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

  - **To find *E(s)*, the error between the input and the output, we can write,**

$$E(s) = input - output$$
$$= R(s) - C(s)$$

  - **Simplifying further for *E(s)*, gives**

$$E(s) = R(s) - \frac{G(s)}{1+G(s)}.R(s)$$

$$= R(s)\left[1 - \frac{G(s)}{1+G(s)}\right]$$

$$= R(s)\left[\frac{1+G(s)-G(s)}{1+G(s)}\right]$$

$$= \frac{1}{1+G(s)}.R(s)$$

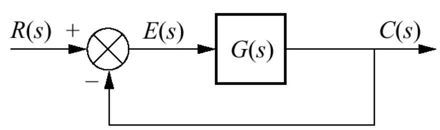
- Using the final value theorem,

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

- In order to simplify the analysis of steady-state error, systems can be classified into <u>System Type.</u>

  - For a unity feedback system as given below,



    with the open loop transfer function,

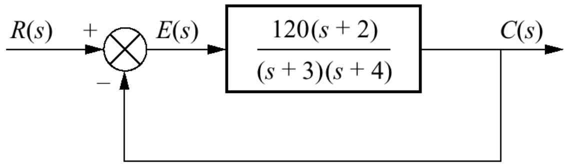
$$G(s) = \frac{K(s+z_1)(s+z_2)...(s+z_m)}{s^N(s+p_1)(s+p_2)...(s+p_n)} \qquad , \quad m \le n$$

    the system type can be determined by identifying the value for *N* at the denominator of the transfer function.

  - For example,

    - If *N* = 0, the system is of Type 0.
    - If *N* = 1, the system is of Type 1.
    - If *N* = 2, the system is of Type 2.
    - And so on.

- **Example:**



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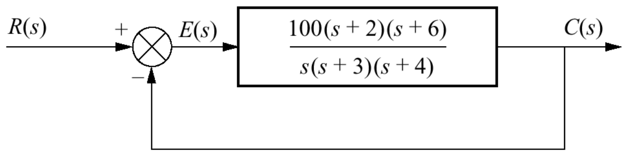

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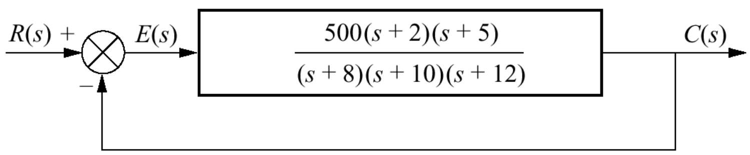
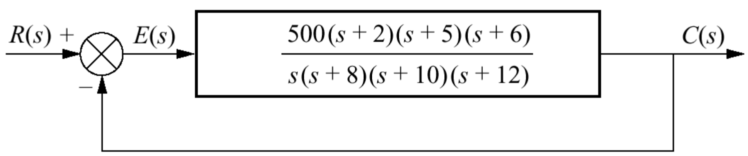
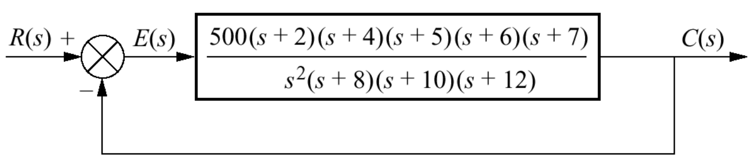


(*a*)



(*b*)



(*c*)

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- $G(s) = \dfrac{1}{s^2 + 4s + 7}$

- $G(s) = \dfrac{(s+1)}{s^3 + 4s^2 + 7s}$