- **The steady-state error for a <u>unity feedback</u> system can be determined easier by identifying the system type and input signal;**

  - **Steady-state error for unit step input:**

$$e_{ss} = \lim_{s \to 0} \frac{s}{1 + G(s)} \cdot \frac{1}{s}$$

$$= \frac{1}{1 + \lim_{s \to 0} G(s)}$$

$$= \frac{1}{1 + K_p}$$

where, $K_p = \lim_{s \to 0} G(s)$

= position error constant

For system of type 0,

$$G(s) = \frac{K(s + z_1)(s + z_2)...(s + z_m)}{(s + p_1)(s + p_2)...(s + p_n)}$$

$$K_p = \lim_{s \to 0} \frac{K(s + z_1)(s + z_2)...(s + z_m)}{(s + p_1)(s + p_2)...(s + p_n)}$$

$$= \frac{K.z_1.z_2.....z_m}{p_1.p_2.....p_n}$$

$$\therefore \quad e_{ss} = \frac{1}{1 + K_p}$$

For system of type 1 and above,

$$G(s) = \frac{K(s + z_1)(s + z_2)...(s + z_m)}{s^N (s + p_1)(s + p_2)...(s + p_n)} \qquad for \quad N \geq 1$$

$$K_p = \lim_{s \to 0} \frac{K(s + z_1)(s + z_2)...(s + z_m)}{s^N (s + p_1)(s + p_2)...(s + p_n)}$$

$$= \infty$$

$$\therefore \quad e_{ss} = \frac{1}{1 + K_p} = 0$$

➔ **Those systems from type 1 and above will have zero steady-state error for step input.**

- **Steady-state error for unit ramp input:**

$$e_{ss} = \lim_{s\to 0} \frac{s}{1+G(s)}.\frac{1}{s^2}$$

$$= \lim_{s\to 0} \frac{1}{s + sG(s)}$$

$$= \frac{1}{\lim_{s\to 0} sG(s)}$$

$$= \frac{1}{K_v}$$

where, $K_v = \lim_{s\to 0} sG(s)$

= *velocity error constant*

For system of type 0,

$$G(s) = \frac{K(s+z_1)(s+z_2)...(s+z_m)}{(s+p_1)(s+p_2)...(s+p_n)}$$

$$K_v = \lim_{s\to 0} s\frac{K(s+z_1)(s+z_2)...(s+z_m)}{(s+p_1)(s+p_2)...(s+p_n)}$$

$$= 0$$

$$\therefore \quad e_{ss} = \frac{1}{K_v} = \infty$$

For system of type 1,

$$G(s) = \frac{K(s+z_1)(s+z_2)...(s+z_m)}{s(s+p_1)(s+p_2)...(s+p_n)}$$

$$K_v = \lim_{s\to 0} s \frac{K(s+z_1)(s+z_2)...(s+z_m)}{s(s+p_1)(s+p_2)...(s+p_n)}$$

$$= \frac{K.z_1.z_2.....z_m}{p_1.p_2.....p_n}$$

$$\therefore \quad e_{ss} = \frac{1}{K_v}$$

For system of type 2 and above,

$$G(s) = \frac{K(s+z_1)(s+z_2)...(s+z_m)}{s^N(s+p_1)(s+p_2)...(s+p_n)} \qquad for \quad N \geq 2$$

$$K_v = \lim_{s\to 0} s \frac{K(s+z_1)(s+z_2)...(s+z_m)}{s^N(s+p_1)(s+p_2)...(s+p_n)}$$

$$= \infty$$

$$\therefore \quad e_{ss} = \frac{1}{K_v} = 0$$

- ➔ **Systems of type 0 will have infinity steady-state error.**
- ➔ **Systems of type 1 will have finite steady-state error.**
- ➔ **Those systems from type 2 and above will have zero steady-state error for ramp input.**

- **Steady-state error for parabola input:**

$$e_{ss} = \lim_{s \to 0} \frac{s}{1+G(s)} \cdot \frac{1}{s^3}$$

$$= \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s)}$$

$$= \frac{1}{\lim_{s \to 0} s^2 G(s)}$$

$$= \frac{1}{K_a}$$

where, $K_a = \lim_{s \to 0} s^2 G(s)$

= *acceleration error constant*

For system of type 0 and 1,

$$G(s) = \frac{K(s+z_1)(s+z_2)...(s+z_m)}{s^N (s+p_1)(s+p_2)...(s+p_n)} \qquad \textit{for} \quad N \le 1$$

$$K_a = \lim_{s \to 0} s^2 \frac{K(s+z_1)(s+z_2)...(s+z_m)}{s^N (s+p_1)(s+p_2)...(s+p_n)} \qquad \textit{for} \quad N \le 1$$

$$= 0$$

$$\therefore \quad e_{ss} = \frac{1}{K_a} = \infty$$

For system of type 2,

$$G(s) = \frac{K(s+z_1)(s+z_2)...(s+z_m)}{s^2 (s+p_1)(s+p_2)...(s+p_n)}$$

$$K_a = \lim_{s \to 0} s^2 \frac{K(s+z_1)(s+z_2)...(s+z_m)}{s^2 (s+p_1)(s+p_2)...(s+p_n)}$$

$$= \frac{K.z_1.z_2.....z_m}{p_1.p_2.....p_n}$$

$$\therefore \quad e_{ss} = \frac{1}{K_a}$$

For system of type 3 and above,

$$G(s) = \frac{K(s+z_1)(s+z_2)...(s+z_m)}{s^N(s+p_1)(s+p_2)...(s+p_n)} \qquad for \quad N \geq 3$$

$$K_a = \lim_{s \to 0} s^2 \frac{K(s+z_1)(s+z_2)...(s+z_m)}{s^N(s+p_1)(s+p_2)...(s+p_n)}$$

$$= \infty$$

$$\therefore \quad e_{ss} = \frac{1}{K_a} = 0$$

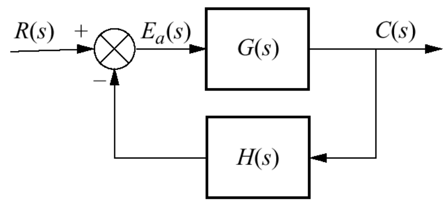
- ➔ **Systems of type 0 and 1 will have infinity steady-state error.**
- ➔ **Systems of type 2 will have finite steady-state error.**
- ➔ **Those systems from type 3 and above will have zero steady-state error for parabolic input.**

- **The steady-state error for a system with <u>unity feedback</u> can be summarized as given below:**

		Type 0		Type 1		Type 2	
Input	Steady-state error formula	Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$

- ***Example:*** **Determine the steady-state errors for the systems in the previous example when they are applied with unit step, unit ramp and unit parabolic inputs.**

- **The steady-state error with <u>non-unity feedback</u> system can be determined in two ways:**



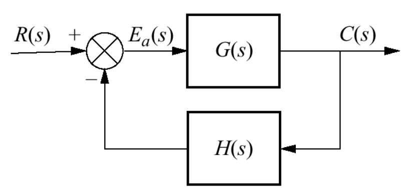
  - **By solving the problem using the fundamental definition of steady-state error,**

$$E(s) = input - output$$
$$= R(s) - C(s)$$

and $$e_{ss} = \lim_{s \to 0} sE(s)$$

  - **By changing the block diagram into the equivalent unity feedback system, and the respective formula to calculate the respective steady-state errors,**

    ie, reducing



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